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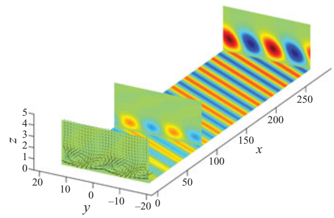
Focus on Fluids

A ‘receptive’ boundary layer



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Receptivity is the process which describes how environmental disturbances (such as gusts, acoustic waves or wall roughness) are filtered by a boundary layer and turned into downstream-growing waves. It is closely related to the identification of initial conditions for the disturbances and requires knowledge of the characteristics of the specific external forcing field. Without such a knowledge, it makes sense to focus on worst case scenarios and search for those initial states which maximize the disturbance amplitude at a given downstream position, and hence to identify upper bounds on growth rates, which will be useful in predicting the transition to turbulence. This philosophical approach has been taken by Tempelmann, Hanifi & Henningson (*J. Fluid Mech.*, 2010, vol. 646, pp. 5–37) in a remarkably complete parametric study of ‘optimal disturbances’ for a model of the flow over a swept wing; they pinpoint the crucial importance both of the spatial variation of the flow and of non-modal disturbances, even when the flow is ‘supercritical’ and hence subject to classical ‘normal mode’ instabilities.

1. Introduction

The viscous equations describing the linear stability of parallel shear flows were presented by Orr and Sommerfeld only a couple of years after Prandtl’s monumental boundary layer theory. Unlike Prandtl’s contribution, which won rapid consensus, the stability theory of parallel and near-parallel flows remained controversial for many years. Indeed, in 1944 C. C. Lin wrote:

> Heisenberg’s remarkable contribution to the hydrodynamic stability of two-dimensional parallel flows has not been favourably accepted and properly appreciated, because his paper is not completely free from obscure points. Nor has the work of Tietjens, Tollmien and Schlichting been properly estimated. As a result, the theory to account for the instability of laminar flow at high Reynolds numbers has become very confused, and its further development has been very much retarded. Various authors suggest that it is necessary (1) to consider disturbances of finite amplitudes, (2) to include the effect of compressibility or even (3) to modify the Navier–Stokes equations.

Over sixty years later, there still remain some ‘obscure points’ in the stability of boundary layers. For example, the base flow which forms on a given body is not invariant along the direction of development of the perturbation x_ϕ (although, in practice, a parallel flow assumption is often tolerable), with x_ϕ itself, along the body surface, not easily identifiable in a generic three-dimensional case (cf. figure 1*a*).

In memoriam: Abdel Zebib, mentor and friend.

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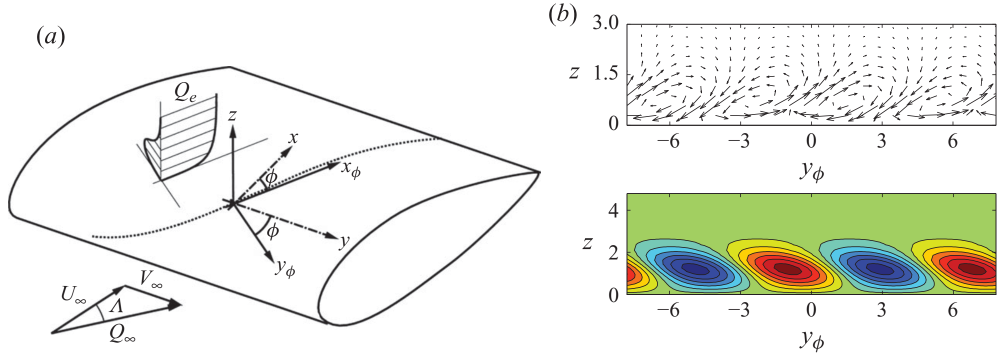

FIGURE 1. (*a*) Boundary layer developing over a swept wing. The path along which disturbances propagate is indicated by a dotted line and coordinates x_ϕ and y_ϕ are attached to this path, while x and y are airfoil-based. (*b*) Optimal disturbances (vector plot) and ensuing streaks (isolines of u_ϕ) in a plane orthogonal to a path of constant phase, for a subcritical adverse-pressure-gradient case.

This spatial evolution of the base flow can be highly significant in the growth and development of perturbations. Furthermore, experiments show that transition to turbulence occurs either via the formation of streaks (elongated regions of high and low values of the downstream velocity component, more or less regularly arranged along a direction orthogonal to x_ϕ) or through the growth and breakdown of wave packets. The occurrence of one event or the other (or indeed a combination of the two) is intimately linked to the environmental conditions, i.e. to the receptivity of the flow to external perturbations.

Classical theory gives little account of the forcing present in the free stream or at the wall, focusing mostly on the threshold for the (quasi-) exponential amplification of monochromatic travelling waves. More recent developments draw heavily on tools from numerical analysis and optimization. These include adjoint equations, non-normal effects and optimal disturbances, concepts which entered the world of hydrodynamic stability about twenty years ago and now are mainstays of the field (Schmid & Henningson 2001). Adjoint fields are related to the receptivity of a flow (cf. Luchini & Bottaro 1998); they are useful in the search for optimal disturbances, defined as states eliciting the largest amplification of the disturbance energy density either over a given time interval or over a fixed distance. In turn, optimals are significant whenever the underlying linear stability operator is highly non-normal.

A simple technique to identify optimals arises after expressing the operator that propagates the inflow condition downstream and consists in searching for its largest singular value (which is the square root of the maximum energy gain). The associated right singular vector is the optimal inflow perturbation, whereas the left singular vector represents the disturbance field ensuing downstream. In the absence of detailed knowledge of the input/boundary forcing fields, the study of worst case scenarios is warranted. Also, large transient growth might have significant implications on transition to turbulence. Finally, it is important to realize that, in the absence of unstable normal modes, the structure of the singular-value spectrum of the propagator may be accountable for driving sub-optimal initial conditions towards output states close to the (first) left singular vector of the propagator, even in the paradoxical case in which optimal inflow states are not physically realizable (Luchini 2000).

By searching for optimals, Tempelmann *et al.* (2010) find preferential receptivity paths for the spatially evolving flow over swept plates. Their work has important implications on the definition of transition thresholds and on laminar flow control. They find that non-modal disturbance amplification can exceed and even supersede the growth of normal modes.

2. Overview

Most studies of optimal disturbances in wall-bounded shear flows deal with the (unphysical) temporal growth of perturbations. This is the case of the local stability analysis of swept boundary layers by Corbett & Bottaro (2001). It is quite speculative to transpose the temporal results to the spatial case, and Corbett & Bottaro conclude their paper by stating that ‘an extension of the current work to the spatial framework is not readily apparent’. Such extension is now available.

A way devised a few years ago to treat weakly non-parallel flows is the so-called ‘parabolized stability equations’ (PSE) approach (Bertolotti, Herbert & Spalart 1992). Rooted in a WKB expansion, the PSE are obtained by decomposing disturbances into a slowly varying shape function S and a wave-like part W, which carries the quasi-exponential growth. The model of Tempelmann *et al.* (2010) reverses this paradigm: the amplification of the perturbation is completely absorbed into S and is assumed to take place only along the chord. The (real) spanwise wavenumber is fixed, and the chordwise wavenumber is prescribed by forcing the perturbation to propagate downstream along lines of constant phase, i.e. paths along the body surface almost aligned with the external streamlines and along which the energy of the wave is transported. The goal of Tempelmann *et al.* is to arrive at a final system which is spatially parabolic, to be able to employ an efficient marching approach to study both modal and non-modal instabilities. The price to pay to obtain a parabolic set of equations is that some chordwise diffusion terms and the chordwise pressure gradient (all related to S) must be neglected, limiting the study to slowly growing waves. The assumptions seem justified since the results closely match those from direct numerical simulations for stationary and travelling crossflow modes.

With the system of equations in place, it becomes relatively ‘easy’ to compute optimal disturbances by numerically integrating the direct and adjoint equations until some optimality condition is satisfied. When the output station is placed in a supercritical region in the space of the parameters, the optimization is straightforward. In fact, if the influence of the environment is completely dominated by the quasi-exponential growth of a mode, only one backward integration of the adjoint equations is sufficient to produce the optimal state at the inflow x_0. The adjoint eigenfunction there is precisely the Green’s function weighting the inflow condition. In subcritical configurations, Tempelmann *et al.* find steady and unsteady optimally growing disturbances with a few iterations of direct and adjoint equations.

In analogy to modal results, unsteady non-modal disturbances grow more than their steady counterparts, for both the accelerated and the decelerated cases, leading to appreciable values of downstream-integrated growth rates not far from the leading edge. However, this does not tell the whole story, since it does not account for the environment. The excitation of travelling waves relies usually on the coupling between a (small) unsteady inflow disturbance and a (small) surface inhomogeneity, and their faster amplification can thus be easily offset by the larger initial amplitude of stationary waves, which thus become the primary culprit for provoking transition.

For example, in-flight turbulence intensities are generally very low and ultimately the nonlinear stage of the transition process is dominated by steady modes.

Tempelmann *et al.* must thus be credited with finding a preferential path for selection and growth of crossflow modes in the realistic spatial framework. The flow structures excited by the transient and the quasi-exponential mechanisms are very similar, so that optimal disturbances display a smooth evolution into unstable crossflow modes. This is different from the case without sweep, where algebraically growing streaks do not smoothly turn into Tollmien–Schlichting waves downstream.

An image of the disturbance fields on a plane orthogonal to the direction of propagation of the perturbation is shown in figure 1(b); another image of transiently growing disturbances, for an accelerated case, is displayed in the figure beside the title. These images convey the strong feeling that the transient mechanism at play is similar to that occurring in two-dimensional boundary layers, with steady vortices at the inflow (characterized by a very small downstream velocity component), which turn into ubiquitous streaky structures at the outflow.

3. Future

Receptivity is the most important factor in defining critical amplitudes for the onset of nonlinear phenomena in boundary layers over swept wings and ultimately for determining the transition 'point'. The results of the theory by Tempelmann *et al.* demonstrate clearly how non-modal growth has the potential to lead to early nonlinear effects and premature transition. Much work will ensue from this theory: as far as receptivity is concerned, two aspects to be studied concern the extreme sensitivity of steady disturbances to the presence of very small surface roughness near the leading edge and the fact that the growth of stationary modes is strongly affected by very weak convex curvature of the wing. Whereas the latter task is within easy reach, the former needs the development of a model which can incorporate the leading edge.

The occurrence of nonlinearities can be treated within the framework developed by Tempelmann *et al.*, possibly coupling the 'modified' PSE to 'conventional' PSE in the large-amplitude range. The ability to conduct nonlinear optimizations is crucial in light of the results by Saric, Carpenter & Reed (2008) showing that discrete roughness elements at the leading edge of a wing can be used to delay the crossflow instability, a step towards the ultimate goal of laminar flow control.

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